Title of paper

Utilizing Leontief's Price Model to Estimate Missing Inventory Data

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Content

Summary

The process of establishing a life cycle inventory can be time consuming. The LCA practitioner is often bound to start form scratch and map all product flows and stressors within the system. Gathering all the desired data is in many cases difficult. We therefore propose an approach that allows to estimate missing inventories using input-output data. We show how the input-output based data is combined with original key data and adapted to represent the input structure of the processes in question. The application of Leontief's price model is essential in doing this. The proposed method allows for a relatively fast approach to establish hybrid LCA inventories under imperfect access to data.

Introduction

The combination of physical life cycle inventories (LCI) with input-output (IO) data has received significant interest within the LCA and Industrial Ecology community in the recent years. Several alternative approaches to combine these two frameworks in various manners have been developed. A thorough summary and discussion of the rational for this and the development of these methods up until 2004 can be found in (Suh.et.al 2004).

We formulate our hybrid LCA system using the notation of input-output analysis (IOA) (Leontief1936). Central to the understanding of the input-output model is the requirements-, or coefficients-, matrix A. The columns of this matrix describe the intermediate inputs a production process requires from itself and other processes, to produce one unit of output. These inputs can be expressed as monetary flows, but also mass and energy flows. The stressor matrix, F, contains the stressor (emission) intensities for each of the processes per unit output, and the C matrix contains characterization or ``equivalency" factors for the various emissions. Following Leontief's tradition, an LCA can be expressed in a single Equation 1. The result is a vector d of category indicator results,

$$d = CF(I - A)^{-1}y \qquad (1)$$

Methodology

In this section we propose a method for estimating missing input-structure data of a set of foreground processes in an hybrid LCA framework. With the term input structure we refer to both requirement of goods and services in addition to the components of value added. Obviously, the more first hand inventory data that is available, the less is left for estimation and thereby increased precision. However, to be able to utilize Leontief's price model (Leontief 1949) to estimate the missing inputs, the prices of the products in the foreground system must be known. Our experience is that getting information on prices is generally less of a challenge than finding data on common inventory items.

The first step in our approach is to identify correspondence between the processes to be modeled by the analyst, denoted *f*, and the aggregated sectors or commodities of the economy, denoted, *n*. This is established in the concordance matrix P_{nf}, consisting of zeros and ones. In addition to the sets *f* and *n* we use *k* to represent the various posts in value added.

Having assigned the processes in our foreground system to various sectors we proceed with extracting the data from the input-output set in order to obtain two matrices $A^{\dagger\dagger}_{nf}$ and $V^{\dagger\dagger}_{nf}$. These contain, respectively, the commodities input- and value added structure of the sectors corresponding to the individual foreground processes. The units of the input structures are converted by multiplying with the price in monetary unit per physical unit of the respective process. Formally we have

$$A^{\dagger\dagger}_{nf} = A_{nn} P_{nf} \hat{p}_f \qquad V'^{\dagger\dagger}_{fk} = V'_{nk} P_{nf} \hat{p}_f \tag{2}$$

We then introduce the sparse matrices of originally gathered key data on the foreground processes, A^*_{nf} and V^*_{fk}. This data will replace the coefficients in $A^{\dagger\dagger}_{nf}$ and $V^{\dagger\dagger}_{nf}$ of identical location. The positions that contain process specific data are therefore identified and stored in the binary matrices θ_{nf} and θ_{fk} which in a given entry contains a one if no original data exists and a zero if original data exists. Having established this we can proceed by removing the elements in $A^{\dagger\dagger}_{nf}$ and $V^{\dagger\dagger}_{nf}$ that we have original data on. The operator used represents element-wise array multiplication.

$$A_{nf}^{\dagger\dagger\dagger} = \Theta_{nf} \odot A_{nf}^{\dagger\dagger} \qquad V_{fk}^{\dagger\dagger\dagger} = \Theta_{fk} \odot V_{fk}^{\dagger\dagger} \tag{3}$$

The $A^{\dagger\dagger\dagger}{}_{nf}$ and $V^{\dagger\dagger\dagger}{}_{nf}$ matrices, now only contain elements that we do not have original data on. We can then proceed to establish the model for the combination of the original data and the estimated data. The A_{nf} and V_{nf} represent the complete matrices that we would like to obtain. These are modeled by adding the matrices of original key data (starred), with the input-output derived data. The input-output data is scaled with a vector, γ_f to be determined.

$$A_{nf} = A_{nf}^{*} + A_{nf}^{\dagger\dagger\dagger}\hat{\gamma}_f \qquad V_{fk} = V_{fk}^{*} + \hat{\gamma}_f V_{fk}^{\dagger\dagger\dagger} \tag{4}$$

In order to estimate the missing inventory items, it is necessary to determine the scaling vector γ_f. To do so we introduce the Leontief price model (Leontief 1949).

$$\begin{bmatrix} p_f \\ p_n \end{bmatrix} = \begin{bmatrix} A'_{ff} & A'_{nf} \\ 0 & A'_{nn} \end{bmatrix} \begin{bmatrix} p_f \\ p_n \end{bmatrix} + \begin{bmatrix} V_{fk}i \\ V_{nk}i \end{bmatrix} \tag{5}$$

We proceed by Inserting Equation 4 into Equation 5. Having the p_f vector given exogenously and applying that by definition p_n is a vector of ones (denoted i), this yields:

$$p_f = A'_{ff}p_f + (A_{nf}^{*} + A_{nf}^{\dagger\dagger\dagger}\hat{\gamma}_f)'i + (V_{fk}^{*} + \hat{\gamma}_f V_{fk}^{\dagger\dagger\dagger})i \tag{6}$$

Rearranging and solving for γ_f gives

$$\gamma_f = (\widehat{A_{nf}'^{\dagger\dagger\dagger}i + V_{fk}^{\dagger\dagger\dagger}i})^{-1}[(I - A'_{ff})p_f - A_{nf}'^{*}i - V_{fk}^{*}i] \tag{7}$$

Inserting γ_f into equation 4 allows us to obtain the A_{nf} and V_{nf} matrices which contain the key data supplied and our estimate for the remaining entries. Having determined these the hybrid requirements matrix and the value added matrix can be compiled:

$$A = \begin{bmatrix} A_{ff} & 0 \\ A_{nf} & A_{nn} \end{bmatrix} \quad V = \begin{bmatrix} V_{fk} \\ V_{nk} \end{bmatrix} \tag{8}$$

Discussion and Conclusion

Establishing original inventories is unfortunately not always a straight forward procedure. For various reasons one does occasionally not get access to all the data one would like to have. We have therefore proposed a method for estimating missing inventory items. Our proposed method utilizes knowledge of key LCA inventory data together with product prices to provide an estimate based on input-output data. The application of the Leontief price model in the estimation process ensures that value added and intermediate requirements balance with the prices. In technical terms we have a system that provide valid solutions both in the primal and dual form. The consistent representation of both flows and costs provided, we expect to be useful in eco-efficeicy analysis

References

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