Applying Leontief's Price Model to Estimate Input Structures

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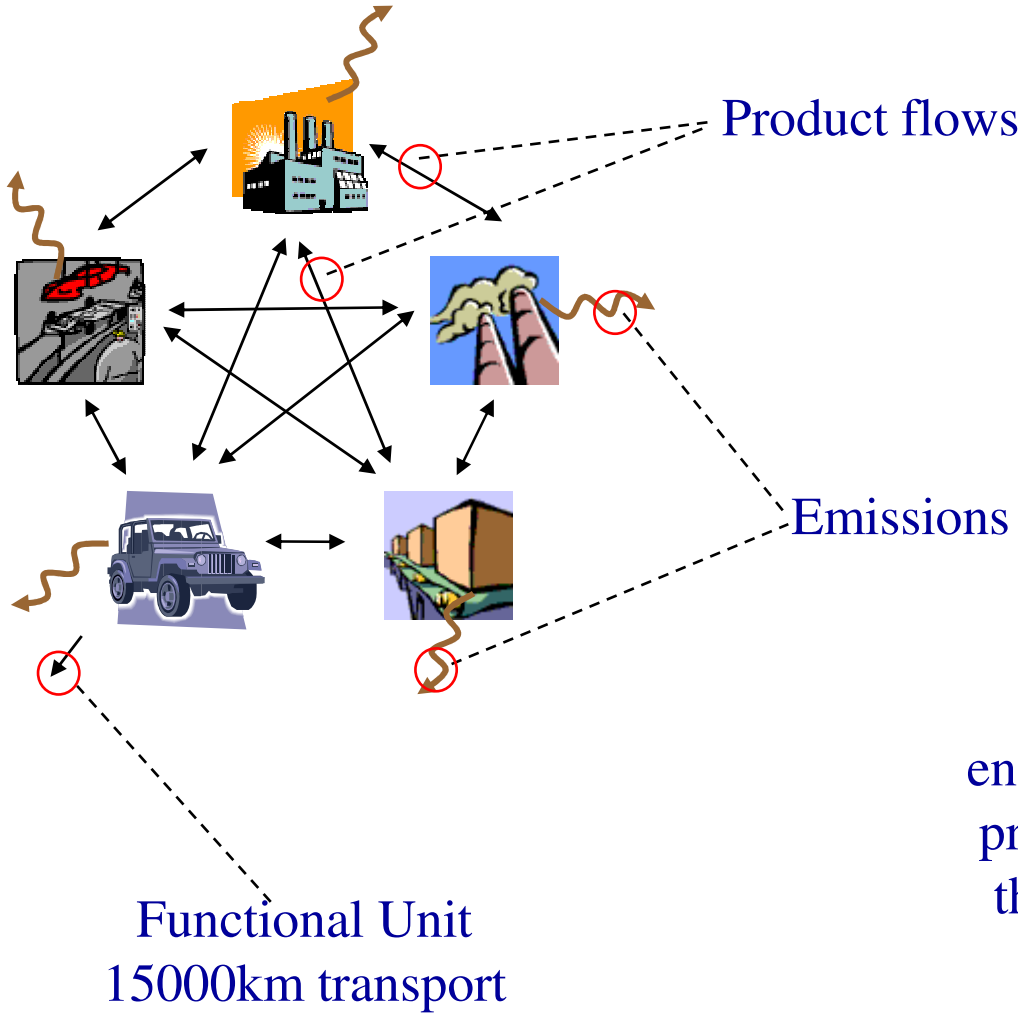
Agenda

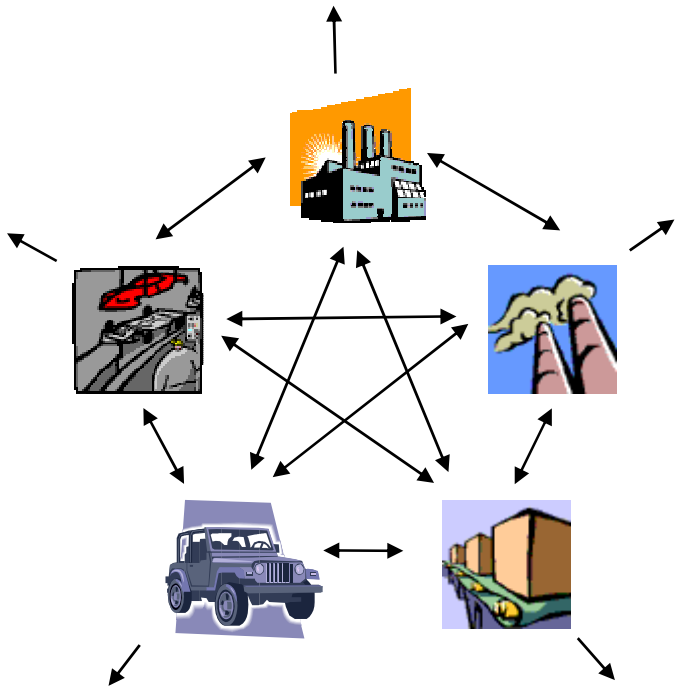
- Introduction and Motivation
- Methodology Development
- Discussion and Conclusion

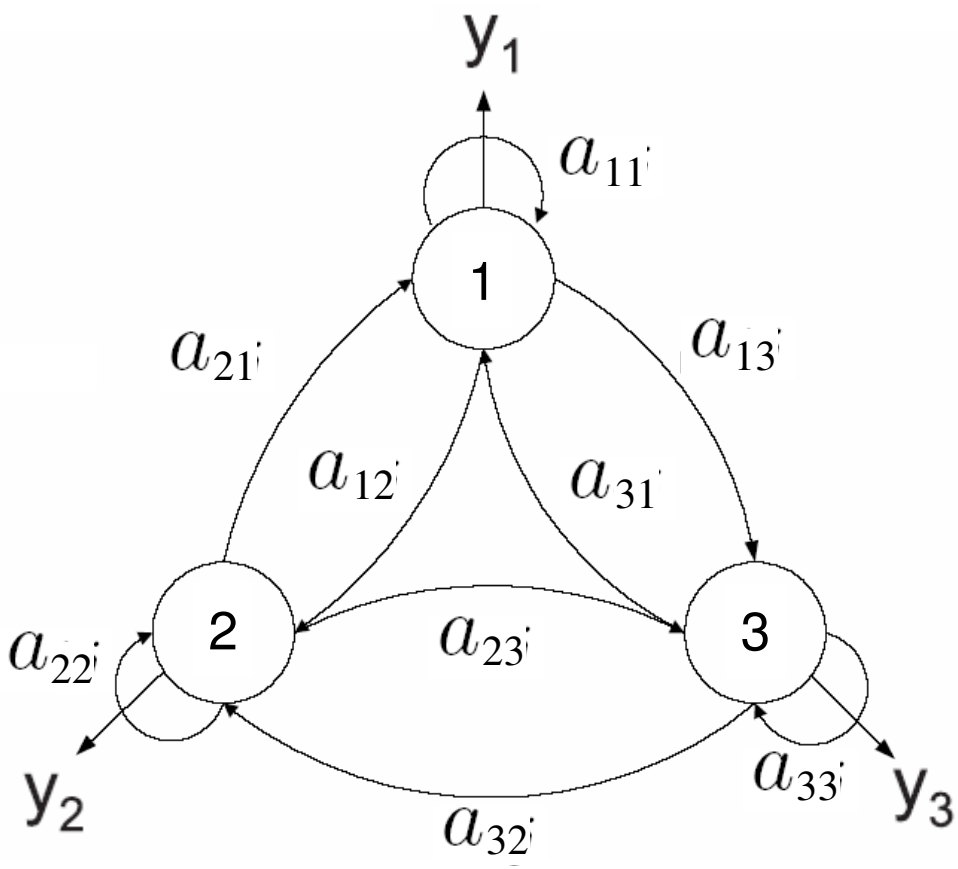
Describe interaction between and environmental stressors originating from relevant processes



Determine total environmental load of the product (functional unit) throughout its life cycle

The open Leontief model

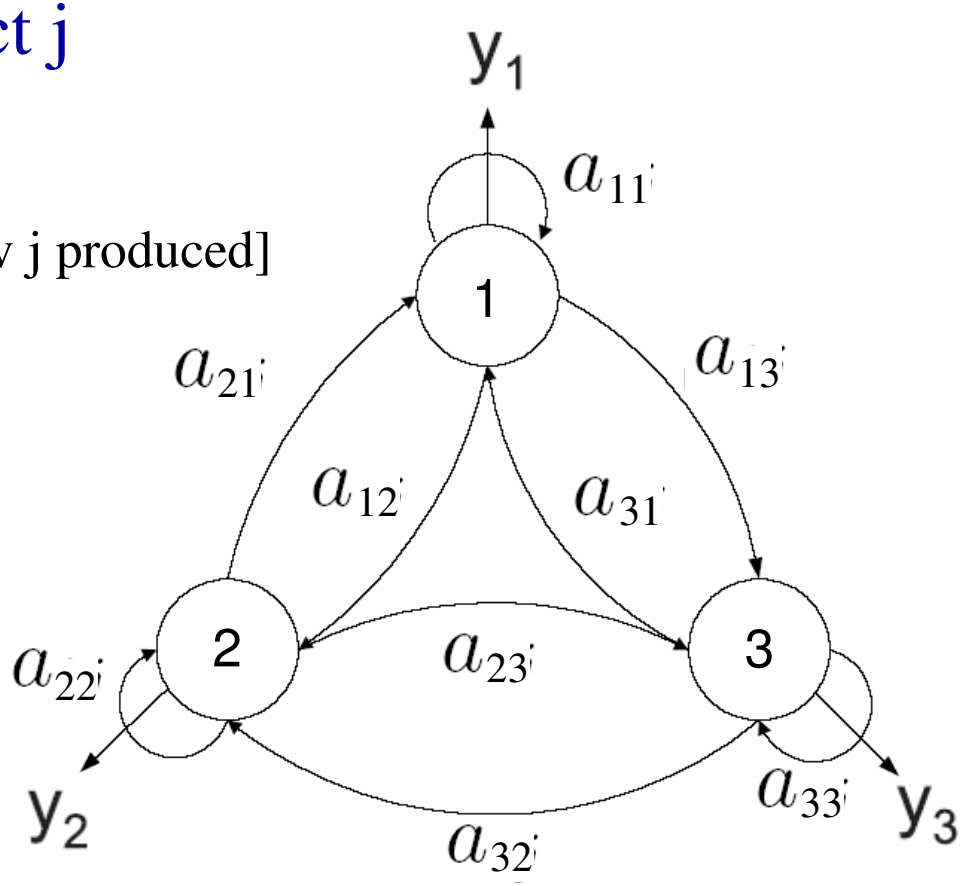




"...the system of economic interrelationships may be represented as a long path describing a wide circle and ending up again at its starting point. All along the way, it branches off into numerous smaller paths, some of which interweave in all manner of combinations..." Leontief (1928).

Cooking recipe for product j

$a_{ij} =$ [kg of i required] / [kg av j produced]



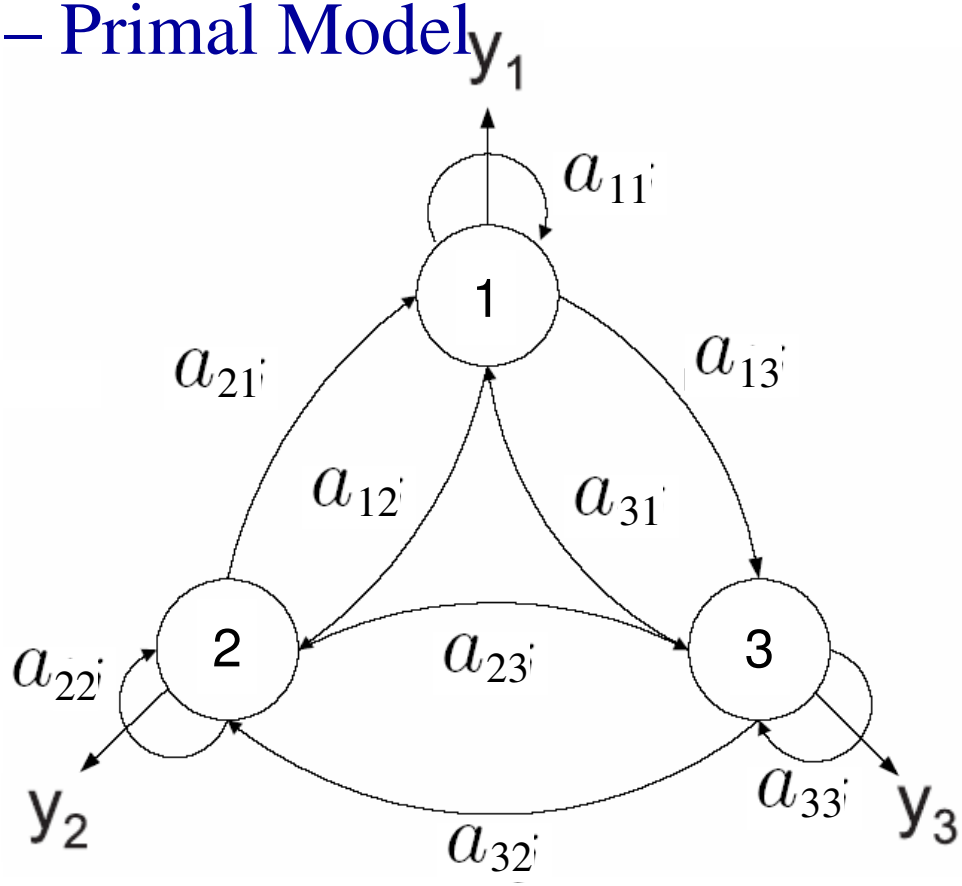
Column 1 in Matrix A
Cooking recipe of product 1

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

The open Leontief Model – Primal Model

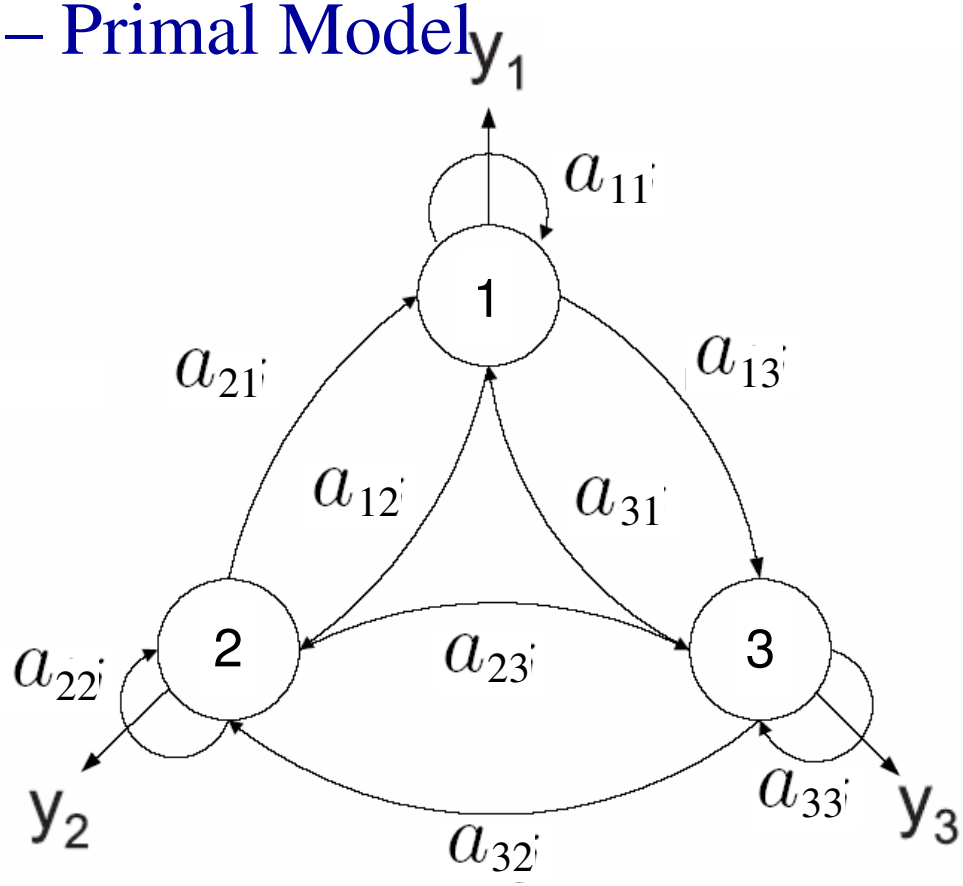
Total Output

$$x = Ax + y$$



$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

The open Leontief Model – Primal Model



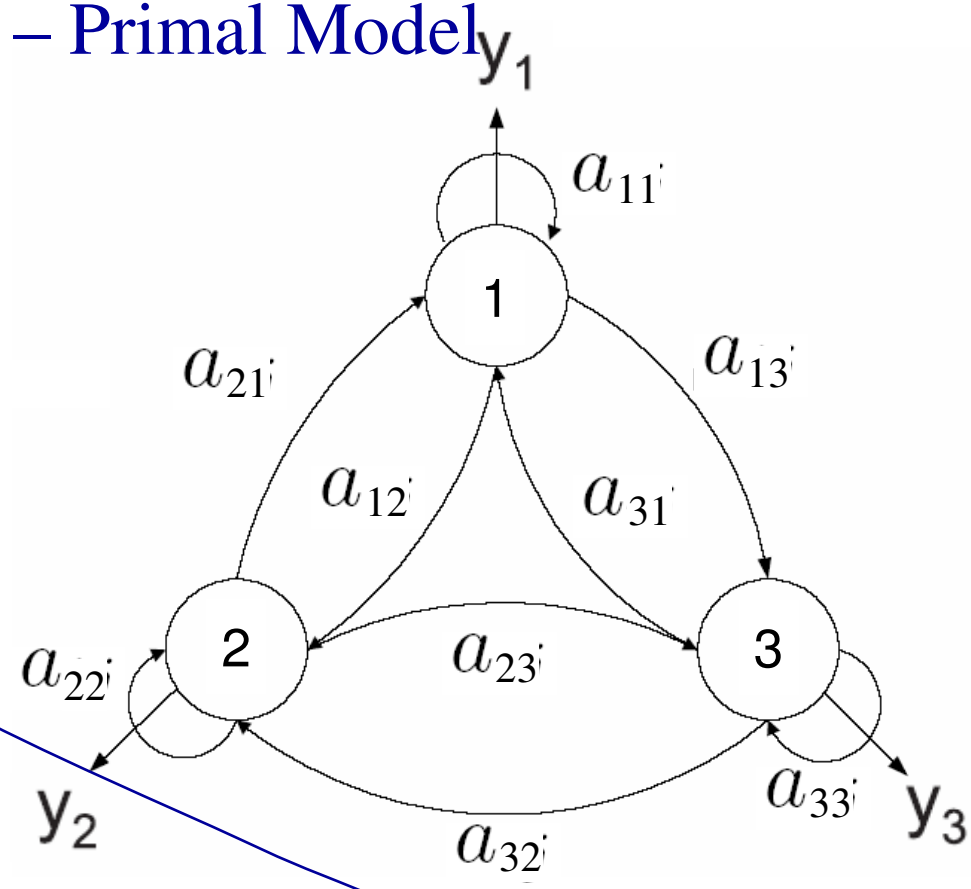
Intermediate Consumption

$$x = Ax + y$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

The open Leontief Model – Primal Model

External demand - Functional Unit



$$x = Ax + y$$

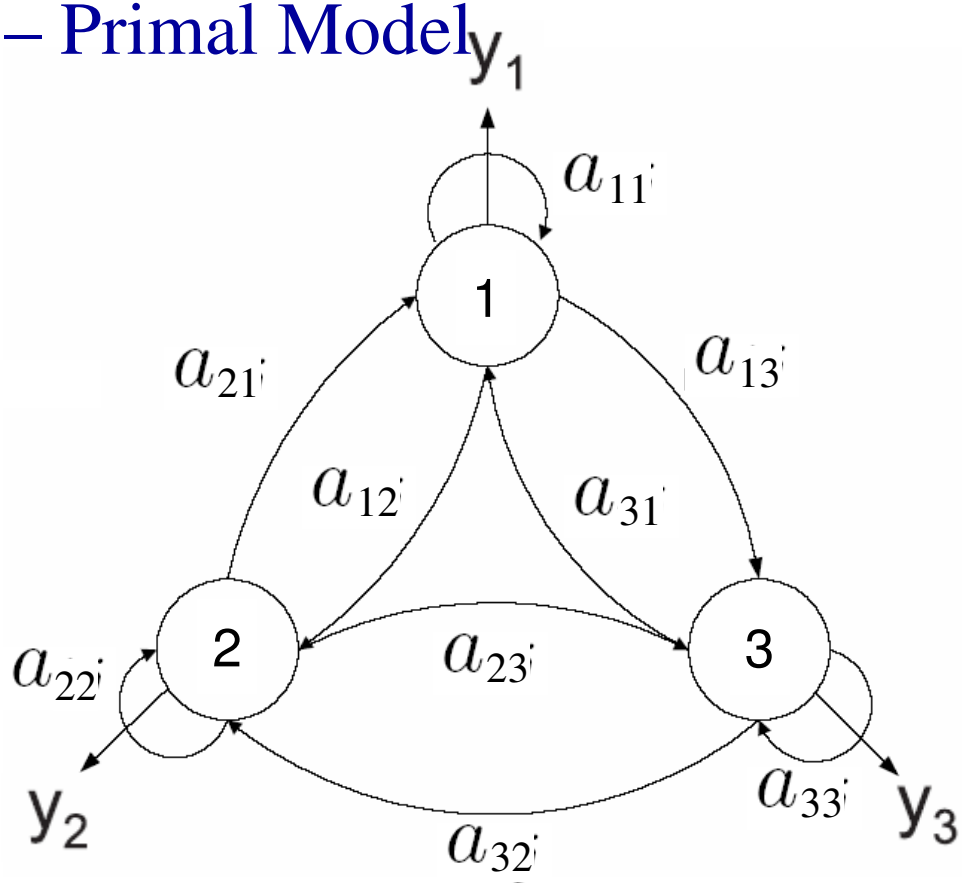
$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

The open Leontief Model – Primal Model

Solving for the output for a given Functional unit

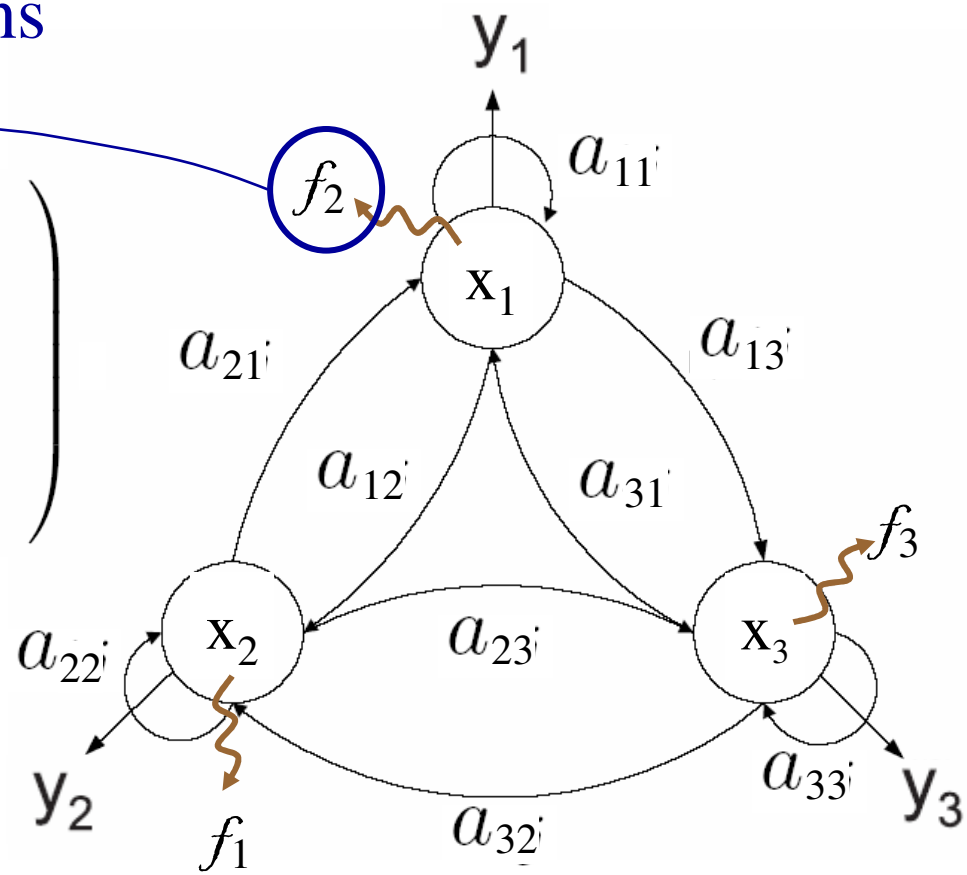
$$x = Ax + y$$

$$x = (I - A)^{-1} y$$



Calculating total Emissions

$$F = \begin{pmatrix} f_{11} & f_{12} & \cdots & f_{1pro} \\ f_{21} & f_{22} & \cdots & f_{2pro} \\ f_{31} & f_{32} & \cdots & f_{3pro} \\ \vdots & \cdots & \ddots & \vdots \\ f_{str1} & f_{str2} & \cdots & f_{strpro} \end{pmatrix}$$



$$x = (I - A)^{-1} y$$

$$\boxed{e = Fx = F(I - A)^{-1} y}$$

[kg CO2] = [kg CO2 /kWh] * [kWh]

[kg CO2 /kWh]
[kg CO2 /kg steel]
[kg NOx /kg concrete]
[kg PAH /tkm transport] etc….

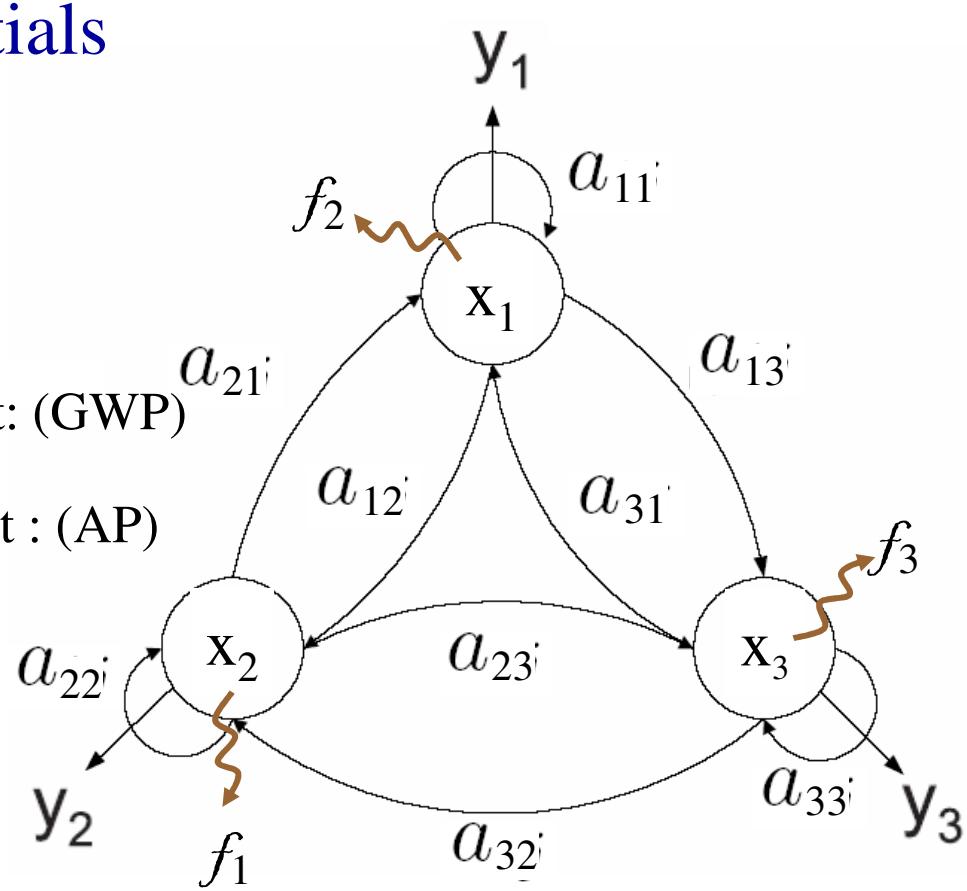
Calculating Impact Potentials

$$C = \begin{array}{cc} \text{kg } CO_2 & \text{kg } NO_x \\ \begin{pmatrix} 1 & 0 \\ 0 & 0.5 \end{pmatrix} & \begin{array}{l} \text{kg } CO_2\text{-eq /subst: (GWP)} \\ \text{kg } SO_2\text{-eq / subst : (AP)} \end{array} \end{array}$$

$$x = (I - A)^{-1} y$$

$$e = Fx = F(I - A)^{-1} y$$

$$\boxed{d = Ce = CF(I - A)^{-1} y}$$



[kg CO2-eq] = [kg CO2-eq /kg CH4] * [kg CH4]

The Primal form of the open Leontief model

$$x_1 = \boxed{a_{11}x_1} + a_{12}x_2 + \ldots + a_{1i}x_i + \ldots + a_{1n}x_n + y_1$$

$$\vdots$$

$$x_i = \boxed{a_{i1}x_1} + a_{i2}x_2 + \ldots + a_{ii}x_i + \ldots + a_{in}x_n + y_i$$

$$\vdots$$

$$x_n = \boxed{a_{n1}x_1} + a_{n2}x_2 + \ldots + a_{ni}x_i + \ldots + a_{nn}x_n + y_n$$

$$x = Ax + y$$

$$x = (I - A)^{-1} y$$

The Dual Form – The Leontief Price Model

$$\boxed{p_1 = a_{11}p_1 + a_{21}p_2 + \ldots + a_{i1}p_i + \ldots + a_{n1}p_n + V_1}$$

$$\vdots$$

$$p_i = a_{1i}p_1 + a_{2i}p_2 + \ldots + a_{ii}p_i + \ldots + a_{ni}p_n + V_i$$

$$\vdots$$

$$p_i = a_{1n}p_1 + a_{2n}p_2 + \ldots + a_{in}p_i + \ldots + a_{nn}p_n + V_n$$

$$p = A'p + V$$

Eco-Efficiency using the Leontief Price Model

- Eco-efficiency = ”added value” / ”environmental impact”
- Eco-efficiency = ”economic value” / ”resource use”
- Eco-efficiency = …

- Finding total impact (Primal Model-Flows):

$$x = Ax + y \qquad d = CF(I - A)^{-1} y$$

- The Leontief Price Model (Dual Model-Costs)

$$p = A'p + V$$

- Eco-Efficiency based on the

- Price $e^{p}_{i,j} = \dfrac{p_j}{d_i}$ - Value added $e^{V}_{i,j} = \dfrac{V_j}{d_i}$

- A consistent formulation of both costs and flows

Challenge and Motivation

- The challenge is to develop inventories for LCA studies that are consistent in both the primal and dual form.
- To make an inventory consistent in the dual form implies a 0% cut-off with respect to costs.
- We propose a method to obtain this in a IO-based Hybrid LCA inventory

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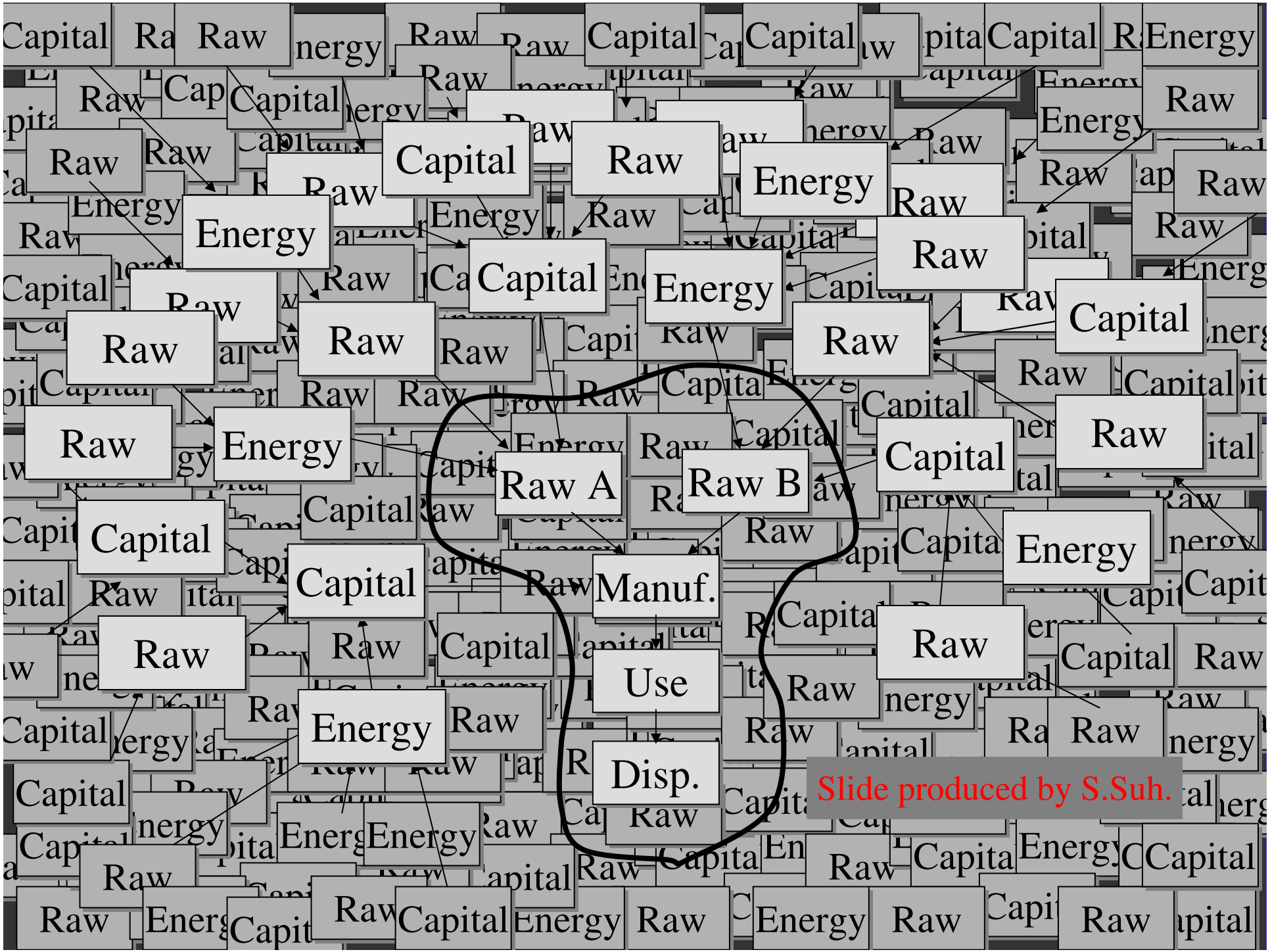
IO-Based Hybrid LCA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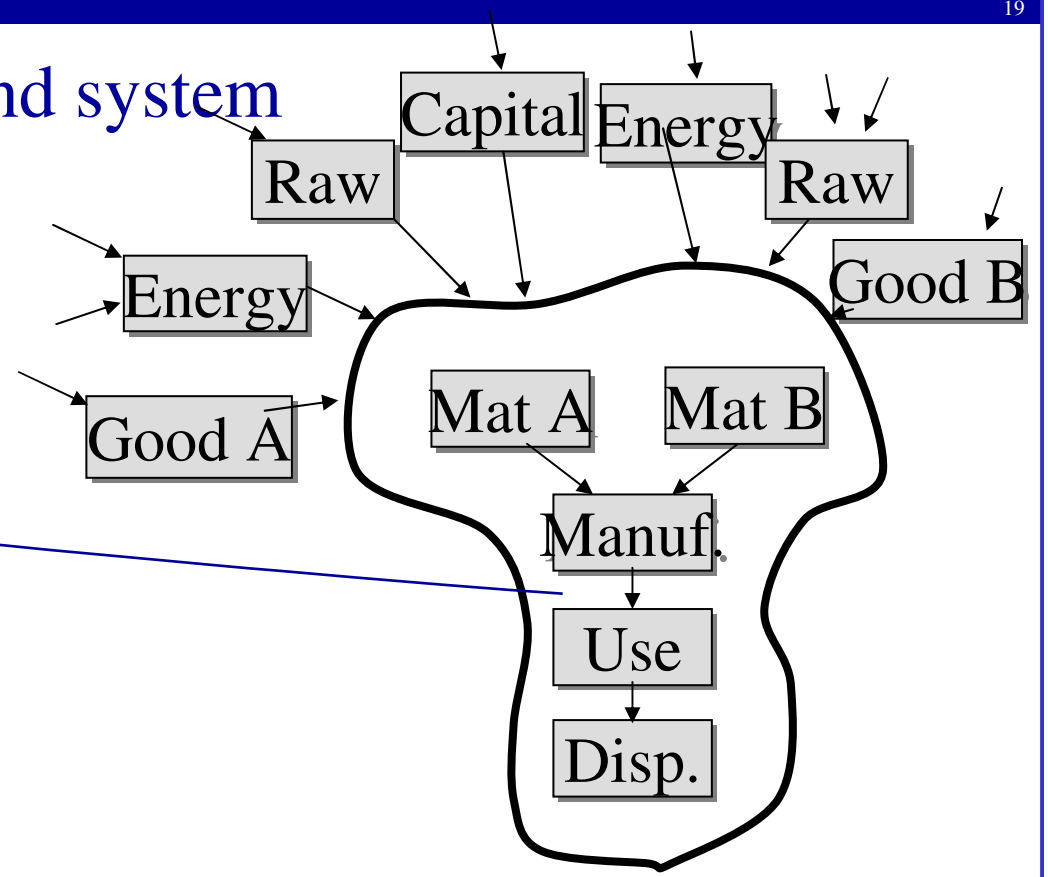
Foreground and background system



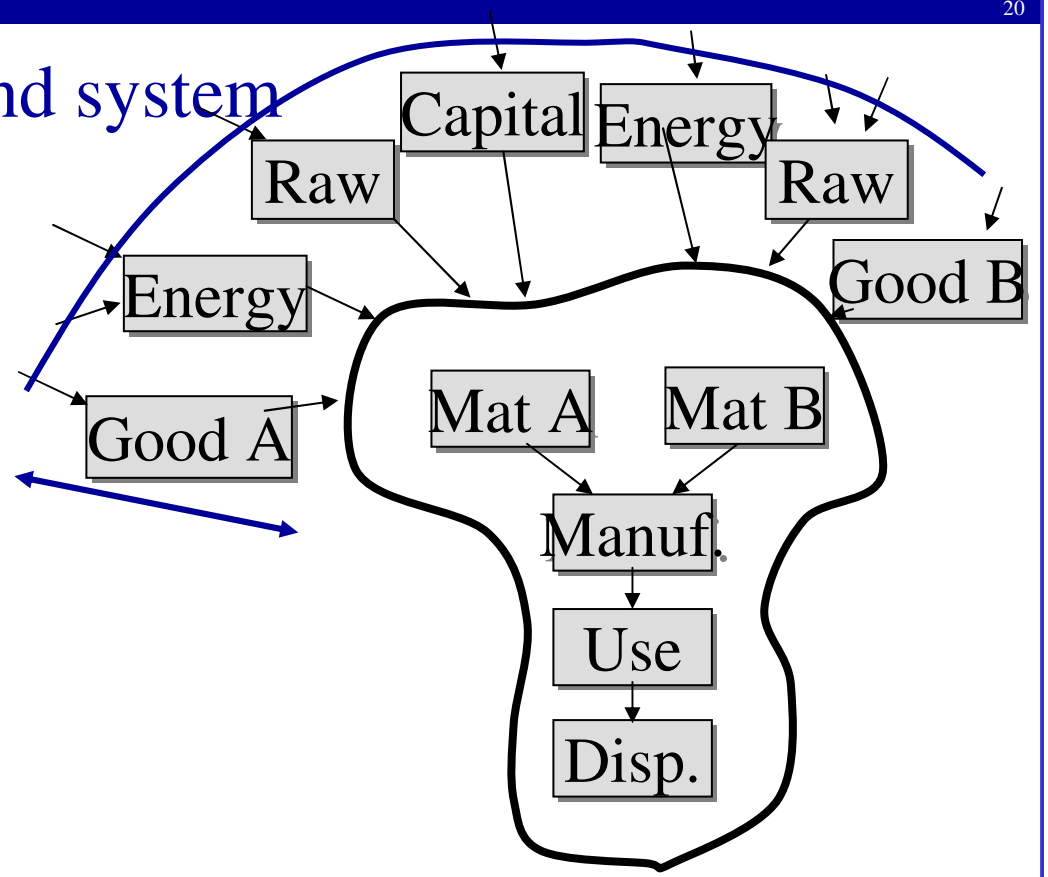
$$A = \begin{bmatrix} \boxed{A_{ff}} & 0 \\ A_{nf} & A_{nn} \end{bmatrix}$$

A_{ff} Foreground processes inter requirements matrix

A_{nn} Background economy coefficients matrix

A_{nf} Upstream inputs of commodities to foreground system

Foreground and background system



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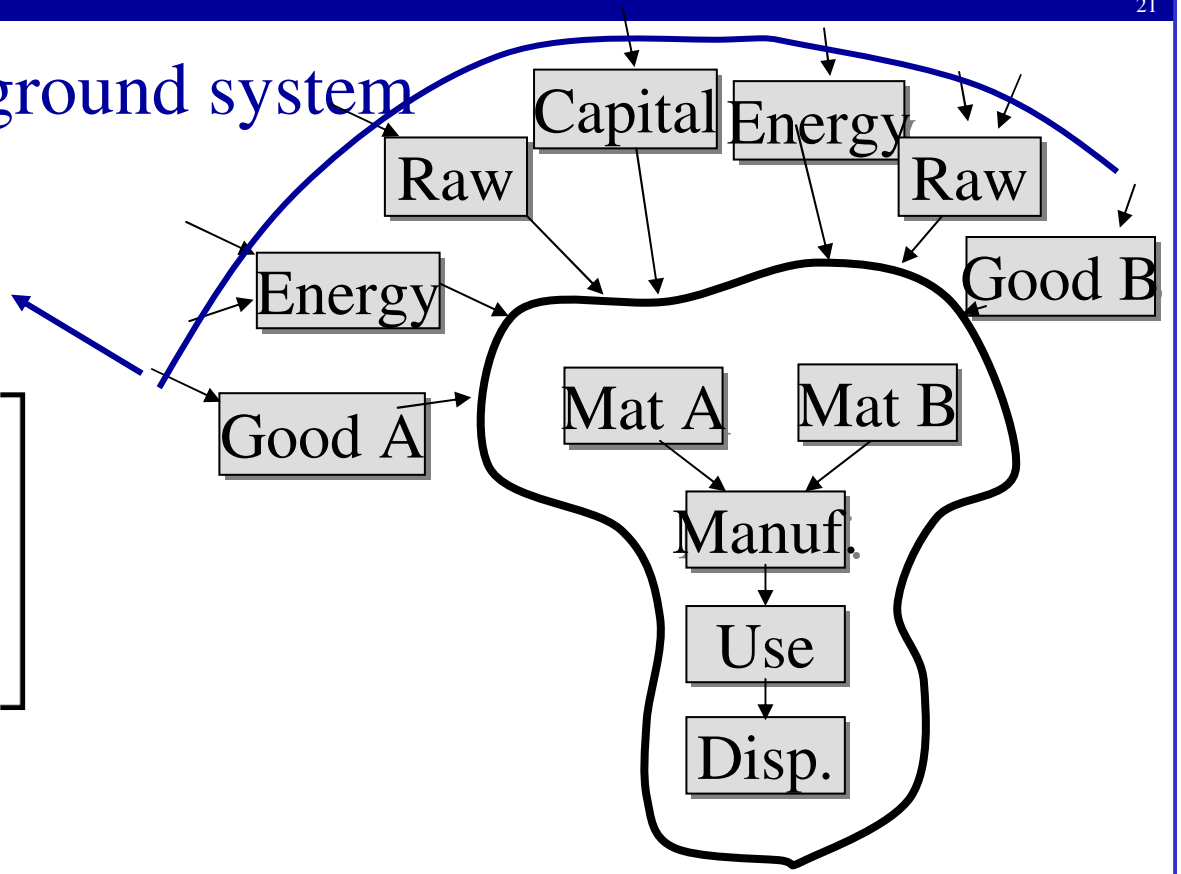

$$A = \begin{bmatrix} A_{ff} & 0 \\ A_{nf} & \boxed{A_{nn}} \end{bmatrix}$$

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Foreground and background system

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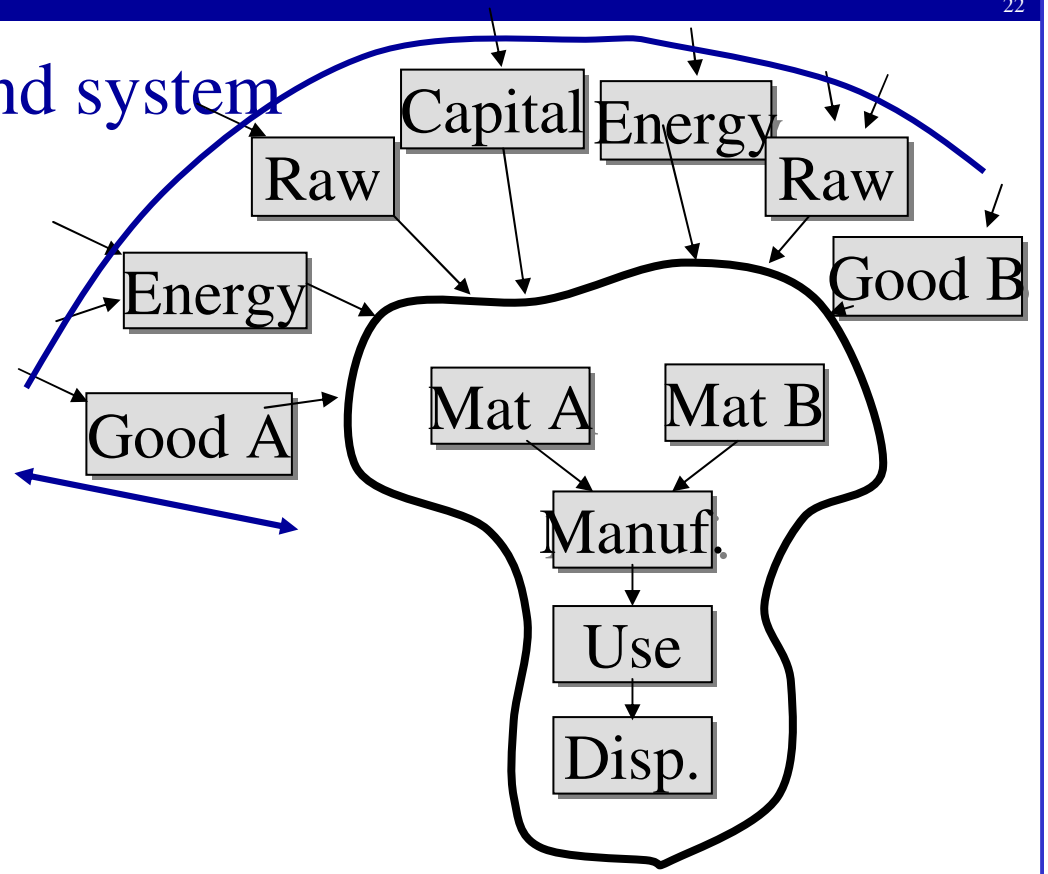
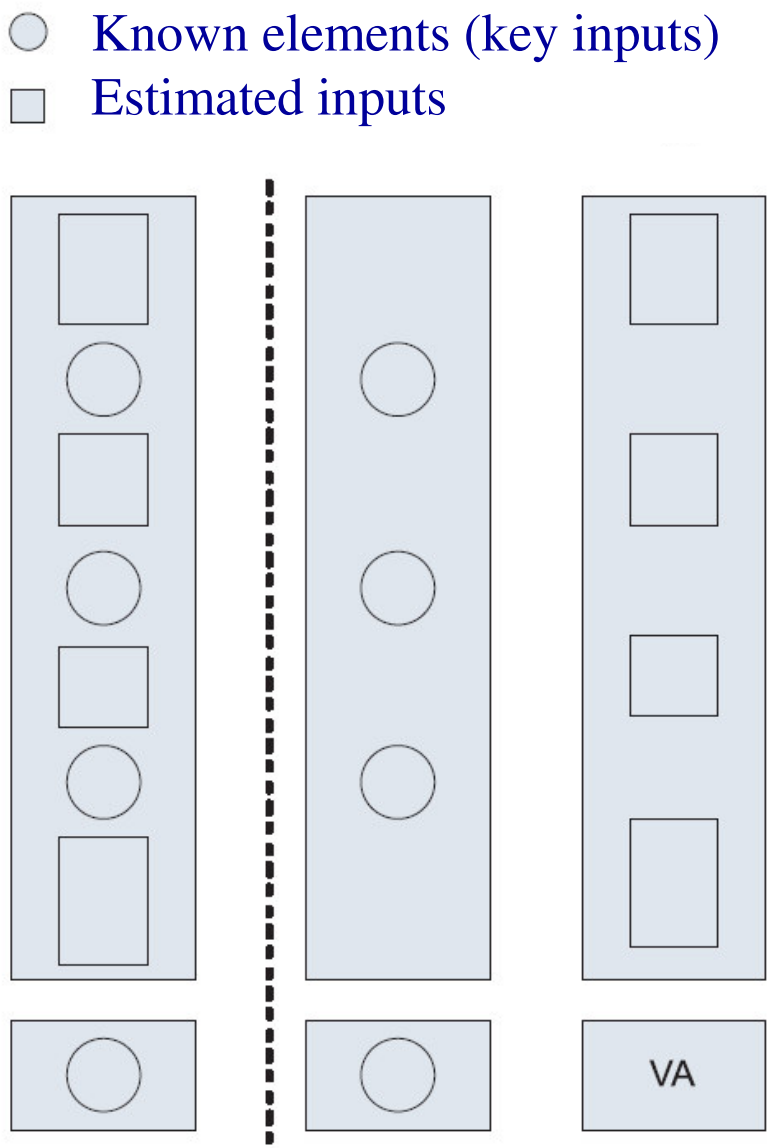


We propose a method for estimating missing elements in the input structure so that the model is valid also in the dual form.

Conceptual Framework

$$A = \begin{bmatrix} A_{ff} & 0 \\ A_{nf} & A_{nn} \end{bmatrix}$$



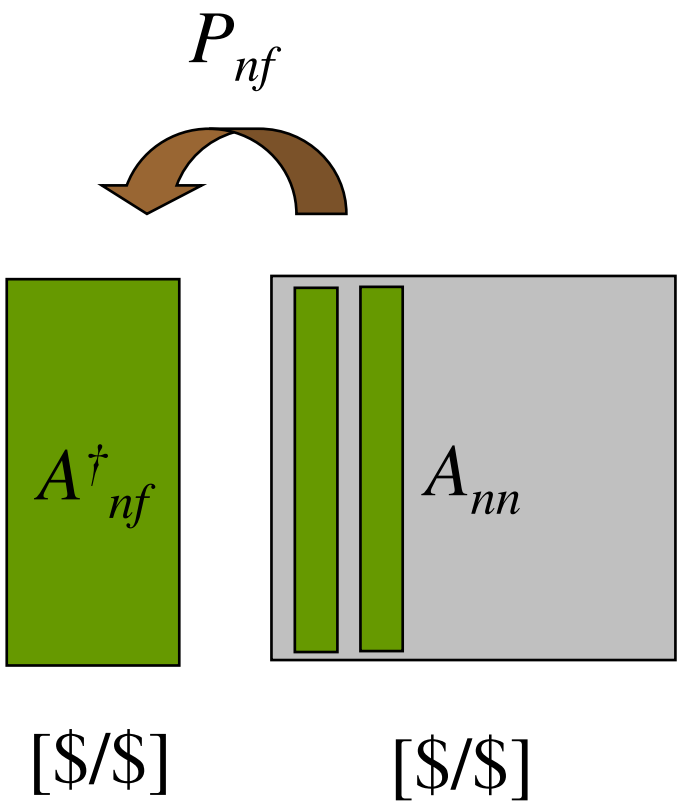
We assume to know the following:

- The internal relations and emission of the foreground system.
- Key inputs form background system
- Prices of goods in the foreground system
- The sector classification of the foreground processes

Adapting input structure from io-data

$$A^{\dagger}_{nf} = A_{nn} P_{nf}$$

$$V'^{\dagger}_{fk} = V'_{nk} P_{nf}$$

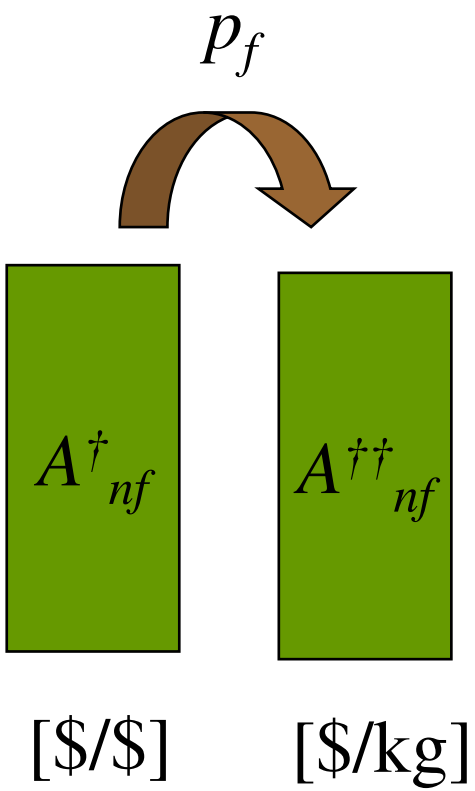


f	Foreground processes. Processes defined in study
n	Background commodities. Commodities of the economy
k	Components of value added

Converting units

$$A^{\dagger\dagger}_{nf} = A^{\dagger}_{nf}\hat{p}_f$$

$$V'^{\dagger\dagger}_{fk} = V'^{\dagger}_{fk}\hat{p}_f$$



f	Foreground processes. Processes defined in study
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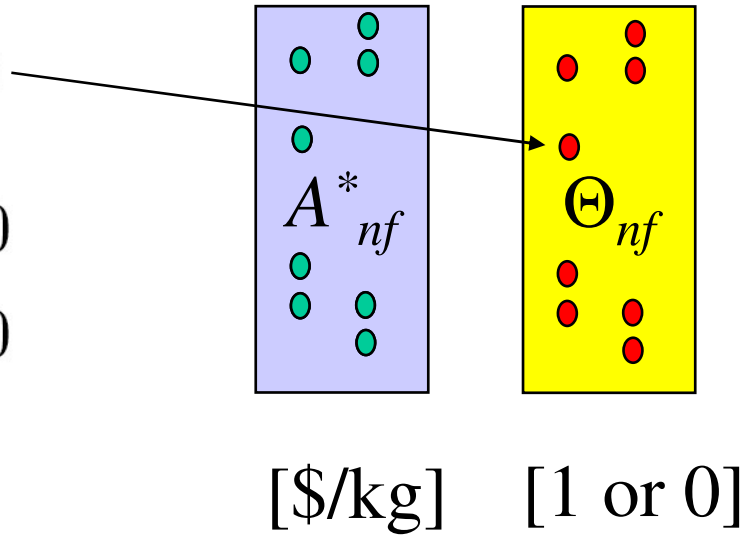
Identifying positions with original key data

$$\Theta_{nf} = \begin{cases} 1 & \text{if} \quad A^*_{nf} = 0 \\ 0 & \text{if} \quad A^*_{nf} \neq 0 \end{cases}$$

$$\Theta_{fk} = \begin{cases} 1 & \text{if} \quad V^*_{fk} = 0 \\ 0 & \text{if} \quad V^*_{fk} \neq 0 \end{cases}$$



f	Foreground processes. Processes defined in study
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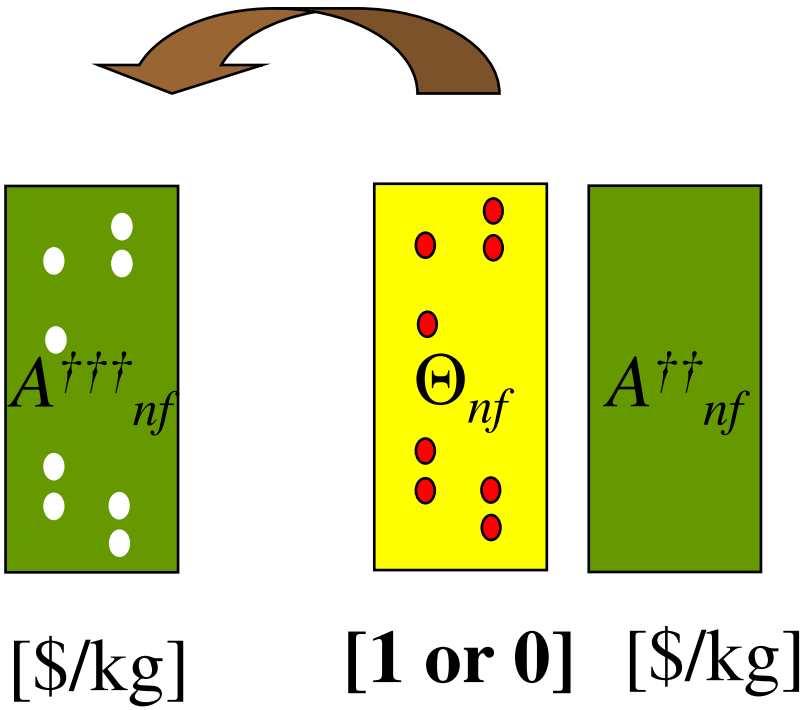
k	Components of value added

Remove elements where original data exists from the io based structure estimate

$$A^{\dagger\dagger\dagger}_{nf} = \Theta_{nf} \odot A^{\dagger\dagger}_{nf}$$

$$V^{\dagger\dagger\dagger}_{fk} = \Theta_{fk} \odot V^{\dagger\dagger}_{fk}$$

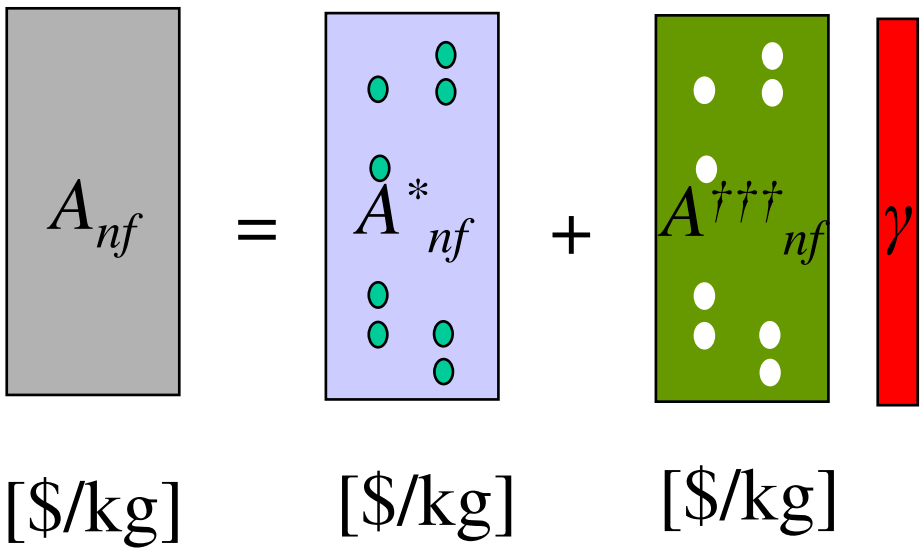


f	Foreground processes. Processes defined in study
n	Background commodities. Commodities of the economy
k	Components of value added

Introduce model for our complete input-structure

$$A_{nf} = A^{*}_{nf} + A^{\dagger\dagger\dagger}_{nf}\hat{\gamma}_f$$

$$V_{fk} = V^{*}_{fk} + \hat{\gamma}_f V^{\dagger\dagger\dagger}_{fk}$$



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$$A_{nf} = A^*_{nf} + A^{\dagger\dagger\dagger}_{nf}\hat{\gamma}_f$$

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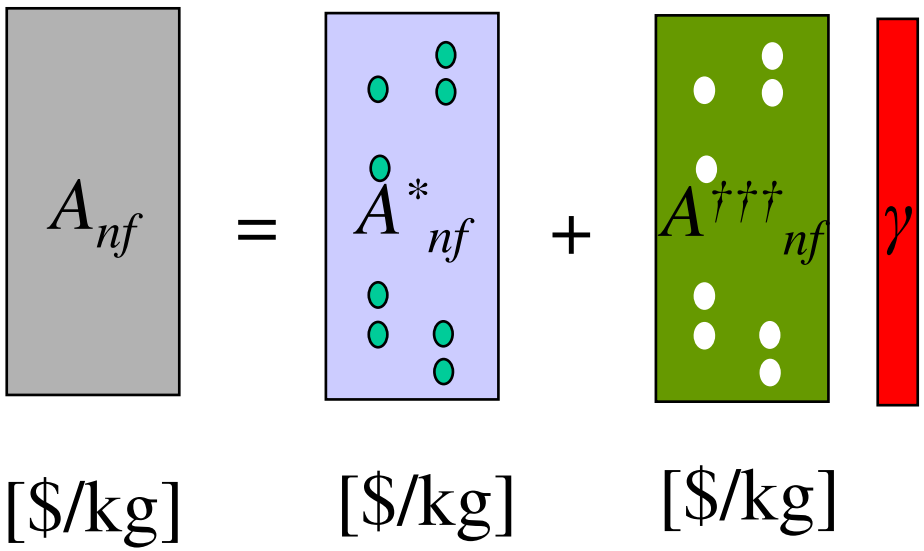
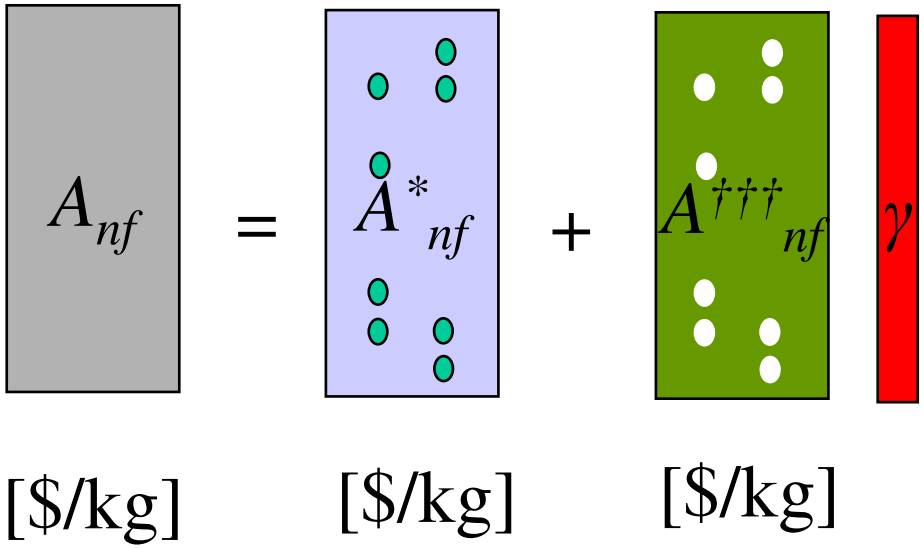


$$A = \begin{bmatrix} A_{ff} & 0 \\ A_{nf} & A_{nn} \end{bmatrix} \qquad V = \begin{bmatrix} V_{fk} \\ V_{nk} \end{bmatrix}$$

Knowing the prices, we apply the leontief price model to scale the inputs

$$A_{nf} = A^*_{nf} + A^{\dagger\dagger\dagger}_{nf}\hat{\gamma}_f$$

$$V_{fk} = V^*_{fk} + \hat{\gamma}_f V^{\dagger\dagger\dagger}_{fk}$$



- Dual – The Leontief Price model

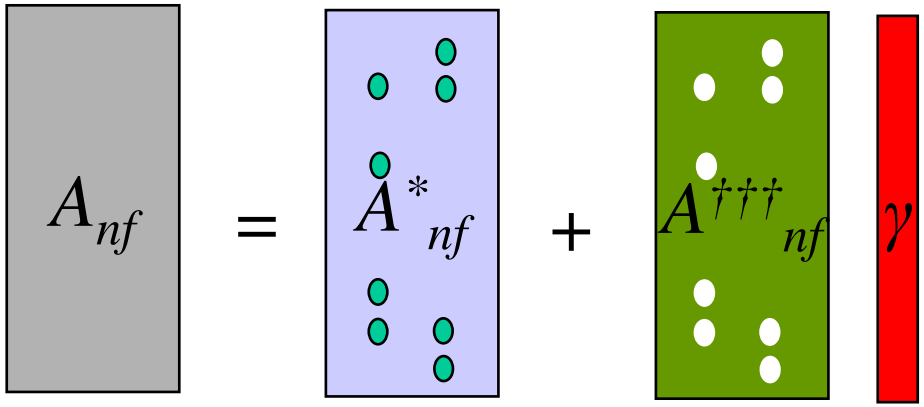
$$\begin{bmatrix} p_f \\ p_n \end{bmatrix} = \begin{bmatrix} A'_{ff} & A'_{nf} \\ 0 & A'_{nn} \end{bmatrix} \begin{bmatrix} p_f \\ p_n \end{bmatrix} + \begin{bmatrix} V_{fk} i \\ V_{nk} i \end{bmatrix}$$

Inserting into the price model

$$p_f = A'_{ff}p_f + (A^*_{nf} + A^{\dagger\dagger\dagger}_{nf}\hat{\gamma}_f)'i + (V^*_{fk} + \hat{\gamma}_f V^{\dagger\dagger\dagger}_{fk})i$$

$$A_{nf} = A^*_{nf} + A^{\dagger\dagger\dagger}_{nf}\hat{\gamma}_f$$

$$V_{fk} = V^*_{fk} + \hat{\gamma}_f V^{\dagger\dagger\dagger}_{fk}$$



[$/kg] [$/kg] [$/kg]

- Dual – The Leontief Price model

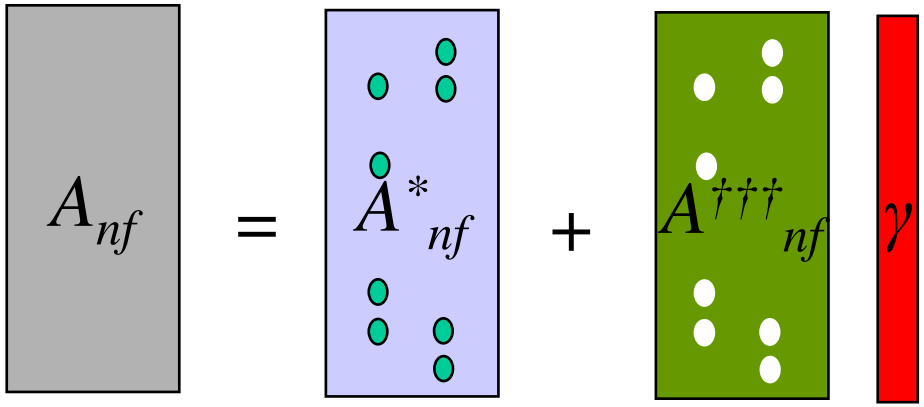
$$\begin{bmatrix} p_f \\ p_n \end{bmatrix} = \begin{bmatrix} A'_{ff} & A'_{nf} \\ 0 & A'_{nn} \end{bmatrix} \begin{bmatrix} p_f \\ p_n \end{bmatrix} + \begin{bmatrix} V_{fk}i \\ V_{nk}i \end{bmatrix}$$

Solving for the scaling vector

$$p_f = A'_{ff} p_f + (A^*_{nf} + A^{\dagger\dagger\dagger}_{nf} \hat{\gamma}_f)' i + (V^*_{fk} + \hat{\gamma}_f V^{\dagger\dagger\dagger}_{fk}) i$$

$$A_{nf} = A^*_{nf} + A^{\dagger\dagger\dagger}_{nf} \hat{\gamma}_f$$

$$V_{fk} = V^*_{fk} + \hat{\gamma}_f V^{\dagger\dagger\dagger}_{fk}$$



[$/kg] [$/kg] [$/kg]

- Solving for the scaling vector (gamma)

$$\gamma_f = (\widehat{A'^{\dagger\dagger\dagger}_{nf} i + V^{\dagger\dagger\dagger}_{fk} i})^{-1} [(I - A'_{ff}) p_f - A'^*_{nf} i - V^*_{fk} i]$$

We have then a description of our production network that is consistent in both the primal and dual form.

- Primal

$$\begin{bmatrix} x_f \\ x_n \end{bmatrix} = \begin{bmatrix} A_{ff} & 0 \\ A_{nf} & A_{nn} \end{bmatrix} \begin{bmatrix} x_f \\ x_n \end{bmatrix} + \begin{bmatrix} y_f \\ y_n \end{bmatrix}$$

- Dual – The Leontief Price model

$$\begin{bmatrix} p_f \\ p_n \end{bmatrix} = \begin{bmatrix} A'_{ff} & A'_{nf} \\ 0 & A'_{nn} \end{bmatrix} \begin{bmatrix} p_f \\ p_n \end{bmatrix} + \begin{bmatrix} V_{fk} i \\ V_{nk} i \end{bmatrix}$$

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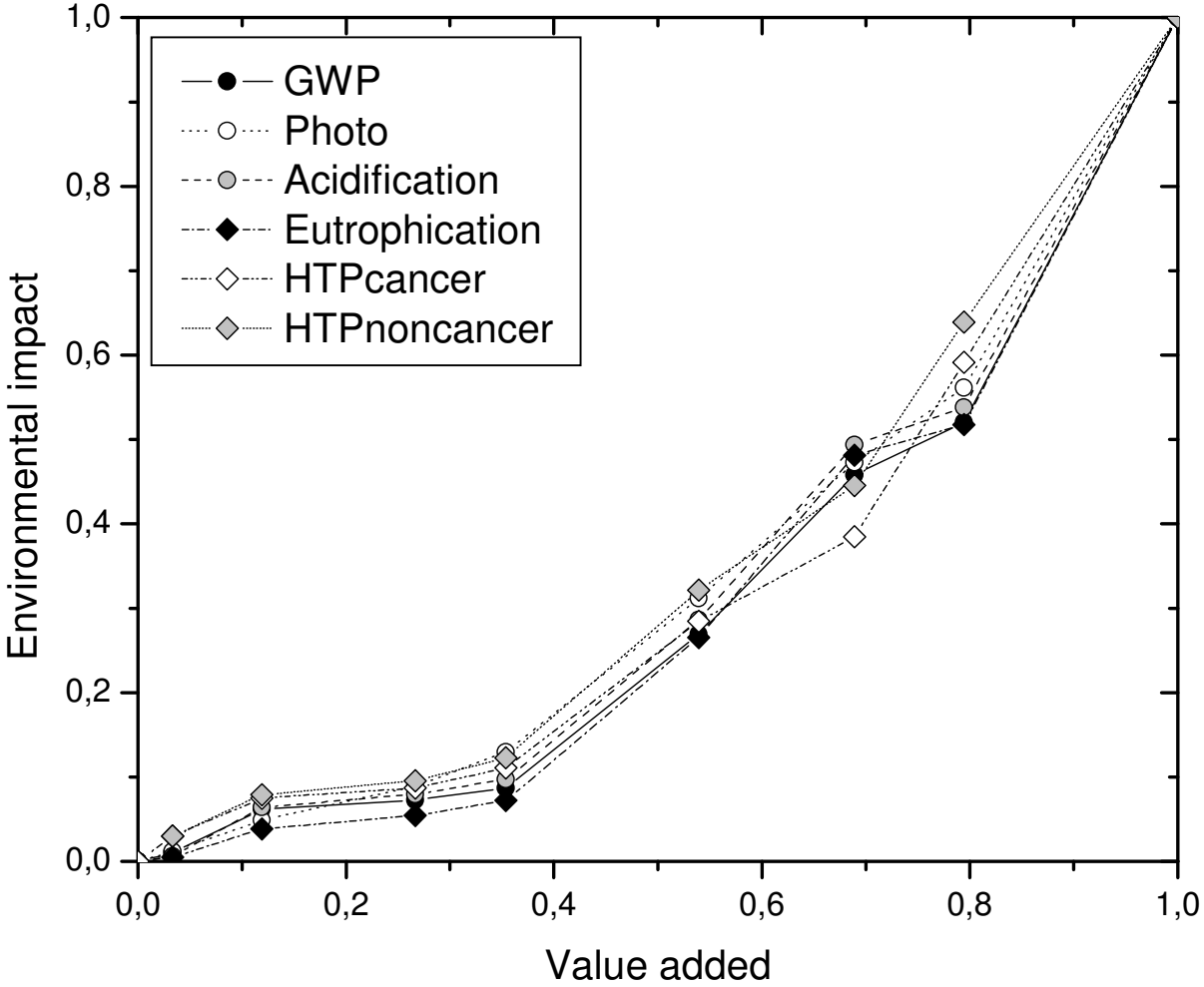
Summary & Conclusion

- Establishing original inventories is unfortunately not always a straight forward procedure. For various reasons one does occasionally not get access to all the data one would like to have.
- We have therefore proposed a method for estimating missing inventory items.
- Our proposed method utilizes knowledge of key LCA inventory data together with product prices to provide an estimate based on input-output data.
- The application of the Leontief price model in the estimation process ensures that value added and intermediate requirements balance with the prices.
- This implies a 0% cut-off with respect to costs
- In technical terms we have a system that provide valid solutions both in the primal and dual form.
- The consistent representation of both flows and costs provided, we have found to be useful in eco-efficiency analysis.

Environmental impact and added value in forestry operations in Norway. Michelsen et.al (2006)